PRIVATE EQUITY MATHEMATICS

SECOND EDITION

Applied analytics and quantitative methods
for private equity investing

Edited by
Oliver Gottschalg
HEC Paris and PERACS Private Equity Track Record Analytics

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About the lead editor

Professor Oliver Gottschalg is part of the Strategy Department at HEC School of Management, Paris. He serves as Academic Dean for the TRIUM Global Executive MBA Programme, directs the HEC Private Equity Observatory and teaches courses on private equity, management buyouts, business strategy and entrepreneurship. He holds a Wirtschaftsingenieur Diploma from the University of Karlsruhe, an MBA from Georgia State University and MSc and PhD degrees from INSEAD.

Oliver's current research focuses on the strategic logic and the performance determinants of private equity investments. His work has been published in leading academic journals and in various publications for practitioners, and has been featured over 100 times in the business media (press, radio, television and online) in the past two years. His book, *Private Equity Mathematics*, is one of the bestselling books with PEI Media.

He regularly presents his research at academic conferences and private equity symposia, and serves as an adviser to leading investors in the private equity industry. He repeatedly served as an adviser to policymakers at the national and European levels in the context of the ongoing debate about a possible need for regulation of the private equity industry.

Oliver's company PERACS is the leading provider of standardised independent private equity track record analytics and validation services, currently advising approximately 20 percent of the market of private equity fund managers fundraising worldwide. ☐

Introduction

By Oliver Gottschalg, HEC Paris and PERACS Private Equity Track Record Analytics

Institutional private equity is playing an increasingly central role in business, as an important and well-established component of alternative investments, as a governance structure that enables the financing of thousands of corporate transformation or expansion strategies, and as a key driver of M&A and IPO activity. It is still, however, a relatively young investment class by most standards. It was less than four decades ago that the industry's pioneers, such as Henry Kravis, Martin Dubilier and Joseph Rice, created this investment model and form of governance. The asset class has since gained prominence to the point that it has attained the lofty moniker of 'Capitalism's new king'. Private equity has grown, matured, expanded its global reach and attracted outstanding talent. At the same time, institutional private equity has become an 'industry' in its own right with an increasing level of professionalisation.

It was only a few years ago that many investors still held the belief that investing in private equity was still much of an 'art', rather than a science, when compared to other asset classes. While some artisanal element remains, the private equity industry has over the past decade developed an increasingly sophisticated range of specific and dedicated tools, benchmarks and methods that help both the general partner (GP) and the limited partner (LP) to make the right investment decisions. Being a great artist requires the mastery of tools and methods; the professionalisation of the private equity industry continues to raise the bar for investors with respect to this requirement.

Still, it is striking that the accessibility of knowledge about this asset class remains low when compared to its economic relevance. At its previous peak during the first half of 2007, private equity was responsible for close to 50 percent of global M&A activity, yet a search in the electronic databases of business journals reveals that there are almost ten times more articles written on 'mergers and acquisitions' than on 'private equity'.

For years, many people believed that almost any form of private equity investment was a sure path to outstanding performance. While research shows that this belief has never been warranted, recent economic difficulties made it clear to everyone once again that only skilled investors can expect to reap attractive returns. Private equity remains a relatively opaque asset class with great information asymmetries. This implies that substantial opportunities are available for investors with superior skills and capabilities – often at the expense of the less skilled.

Historically, the spread between the best and the worst investment opportunities has been much greater in private equity than in many other asset classes. Being average has never been an attractive position and only the upper half of the performance spectrum

yielded returns that clearly compensated investors for the risk and the illiquidity characteristic of this type of investment. At the same time, the very best private equity investments have generated an almost unparalleled performance. The recent crisis of 2008–09 not only put pressure on the overall returns of this asset class, but also made the difference between the best and worst private equity investments and investors clearly visible.

This emphasises the need for investors both GPs and LPs alike to equip themselves with the latest and most sophisticated methods and techniques to assess investment opportunities, to value businesses, to benchmark portfolio performance, and to design incentives for executives and fund managers.

This guide, *Private Equity Mathematics, Second Edition*, aims to provide a comprehensive and timely account of the state-of-the-art, available mathematical tools and methods that inform and guide relevant decisions in all aspects of private equity investing. It presents the theoretical background and lays out formulae whenever necessary. At the same time, it has been written in a pragmatic spirit and intends to focus on the question 'How to …?' rather than to expound on the latest abstract theoretical debate around a given concept. As such, most chapters include practical example calculations that can be easily adjusted to the reader's real-world applications. More complex calculations are illustrated and facilitated based on detailed spreadsheet models, which are available to readers on request.

In this edition, the content has been updated and expanded to reflect the latest advancements and thinking in a given area. Several chapters have been added to integrate recent advancements in the analytical approaches to the private equity asset class. Of particular relevance are the updated chapters on performance measurement and benchmarking, along with a new chapter on performance persistence. Further, three chapters are dedicated to the important topic of risk, reflecting the progress made towards its integration into private equity investment considerations.

I would like to extend my thanks to the contributors for sharing insights on their respective areas of expertise. Their investment of time and their willingness to make best practices available is greatly appreciated, as without it, this project would never have been possible. It is my hope that private equity professionals will be able to improve their investment decisions based on the mathematical methods and tools contained within this publication and that this guide further contributes to the advancement of knowledge about this important and expanding asset class.

Chapter overview

The topics in this guide are broadly divided into three sections. The first section, **Fundamentals**, looks at the most relevant distinguishing features of this asset class: performance, cash flow patterns and risk. The second section, **Investing**, focuses on a variety of issues relevant to GPs and LPs alike, from the evaluation of a possible investment opportunity to different aspects of performance benchmarking, the identification of performance drivers and their persistence across time. The third section, **Fund and**

portfolio management, covers the economic and legal aspects of operating a private equity investment house or a private equity investment programme.

Chapter 1, *Private equity as part of your portfolio*, by Satyan Malhotra of Caspian Private Equity, lays the foundations for the first section by providing an overview of relevant risk and return considerations for the construction of a private equity portfolio. The chapter *Measuring private equity performance* by Ludovic Phalippou of the University of Oxford illustrates the dangers of an imprudent application of widely used but not always appropriate performance measures. Ivan Herger of Capital Dynamics extends this discussion to the complexities of modelling net cash flows from private equity investments based on J-curve projections for both primary and secondary fund investments. The following three chapters address questions of risk in private equity, starting with the chapter by Fernando Vazquez of PERACS which provides insights into the ability to measure and benchmark private equity risk profiles for GPs and LPs. Bernd Kreuter of Palladio Partners and Oliver Gottschalg of HEC Paris and PERACS demonstrate a Monte Carlo approach for risk management in private equity portfolios. Elias Korosis of Hermes GPE and Roy Kuo of Church Commissioners round off the risk discussion with their treatment of methods to integrate risk measures into a risk budgeting approach.

The second section on investing starts with a chapter on the quantification of individual drivers of returns of private equity investments by Oliver Gottschalg. Brian Gallagher of Twin Bridge Capital Partners tackles the question of investment valuation from the perspective of a buyout investor. The following three chapters look at complementary methods to benchmark the performance of private equity investments. Robert Ryan of PERACS addresses the challenges of constructing a meaningful benchmark to benchmark one private equity fund to comparable private equity investments. Alexander Peter Groh of EMLYON presents the latest techniques in assessing the risk-adjusted performance of private equity investments based on public market benchmarks, which are complemented by Oliver Gottschalg's pragmatic approach method to estimating the relative performance of private equity investments in the following chapter. This section concludes with Oliver Gottschalg's chapter on the latest findings on performance persistence in private equity, that is, the likelihood of past outperformers to again outperform in the future.

The last section focuses on the management of private equity funds and portfolios. John Barber of Bridgepoint outlines the relevant formulae and nuances of the economics and incentives of running a private equity firm. The following two chapters deal with economic and legal aspects of the management compensation in MBOs. Michael J. Album, Trevor J. Chaplick, and Joshua M. Miller of Proskauer Rose treat the US context, while Jenny Wheater and Pierfrancesco Carbone of Duane Morris look at the same issue for different European jurisdictions. Leon Hadass of Pantheon and Arantxa Prado examine the optimal construction and assessment of a fund of funds portfolio. Michael J. Ryan of Hamilton Lane investigates methods to assess the performance of private equity service providers, and Griffith Norville of Hamilton Lane concludes this section and the book with a discussion of approaches to measuring volatility in private equity. □

1 Private equity as part of your portfolio

By Satyan Malhotra, Caspian Private Equity

Introduction

It is generally agreed on that investment portfolios undergo the classic life cycle of construct, nurture and harvest.[1] Most of the extant research on investing expound on the general principles articulated by Harry Markowitz in his 1952 paper that serves as the foundation of modern portfolio theory (MPT). Markowitz's research assumes that a portfolio is comprised of assets that are, among other things, fungible, transparent, readily quoted and easily transferable. These elements contribute towards understanding the risk-reward trade-offs among investment choices, thereby allowing the portfolio manager to build an appropriate portfolio given his/her individual utility function.

Private equity as an investment option raises unique challenges, including:

- *Construct phase* – lack of unitised/clean data; non-uniform access with generally large minimums, cash flow uncertainty and multi-year commitments; qualitative aspects (for example, talent, relationships) and other such elements.
- *Nurture phase* – lack of ability to actively manage or assert influence could vary from being completely passive for limited partners (LP) to being active for general partners (GP). However, post-portfolio construction (or when making an acquisition), even the most active GPs can do little other than continue to be active in the individual portfolio companies themselves.
- *Harvest phase* – lack of multiple or defined exit options imply realisations could be suboptimal or span many years. The continuing development of the secondary markets, structured products and listed private equity funds notwithstanding, exit options are quite limited which make the asset class illiquid.

Further, the private equity industry as a whole is not known to maintain robust data sets, due to issues such as lack of depth, lag in information, lack of true price discovery, as well as selection and self-reporting biases. Reported returns are not normally distributed and they are also capital weighted, which makes uniform, unitised allocation analysis very difficult. It can also be generally agreed on that possibly the most important aspect of private equity portfolio management is upfront selection, whether an LP making an investment in a GP or an investment a GP makes in a portfolio company.

Therefore, given the uniqueness of private equity, its data issues and the overlay of multiple non-quantifiable elements, private equity portfolio management is as much an art as a science. Even if it is not possible to clearly articulate the exact methods of portfolio management, it may be possible to identify some general parameters, principles and metrics

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[1] Depending on trading or maturity strategies, the portfolios may be with or without composition churn during the holding period.

(herein collectively called 'private equity tools'). The potential application of private equity tools in managing private equity portfolios is unique to the type of participant:

- GP focus on industry sub-sectors[2] (for example, IT, industrial)
- Fund of funds focus on various types of GPs (for example, buyout, secondary)
- LP focus on types of investments (for example, private equity, public equity, fixed income)

This chapter aims to demonstrate methods of estimating private equity metrics as well as highlight illustrations and presentation styles specific to each private equity participant (that is, GPs, funds of funds and investors). We begin by presenting select private equity metrics and then performing sample analyses from the perspective of each private equity participant. At the onset, it is also equally important to remind the readers of the numerous concerns highlighted above; therefore, the results should be used with extreme caution and more so as relative anchor points are used with some degree of freedom.

Private equity metrics

As with all market practitioners, private equity participants have their own preferences about the metrics they use for portfolio management. Although the metrics, exact formulae and their utility may vary across practitioners, the analysis itself can be grouped into three general categories: (1) return-related, (2) risk-related, and (3) at the portfolio level. This section presents select private equity metrics and their estimation formulae for each of the three general categories.

Return-related

Expected return is a mathematical expectation of return from a single holding or portfolio of holdings. It is generally based on the expected probability of each return. In quantifying the expected return, it is important to establish the parameters around the expected return or whether it is: (a) relative or absolute, and (b) cash-on-cash or in percentages (that is, a 2x multiple return is 41 percent IRR if cash is returned in year 2 versus 10 percent IRR if cash is returned in year 7).

Mean return is the arithmetic average of the return. *Weighted average mean return* would include an additional set of information along with the return for the holding (for example, assets, number of holdings, capital invested).

Quartile is the measure of the relative ranking of the holding (for example, return). The Kth quartile of population X can be defined as the value 'x' such that:

$P(X \leq x) \leq p$ and $P(X \geq x) \geq 1 - p$

where:

$$p = \frac{k}{4}, \text{ for } k = 1, \ldots, 4$$

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[2] For the purposes of this chapter, the focus is on the industry sub-sector as a whole, rather than unique opportunities within the sub-sector.

17 | Insights in assessing the performance of private equity service providers

By Michael J. Ryan, Hamilton Lane

In search of a roadmap

Since the mid-1990s, private equity investing has matured dramatically, evolving from its beginnings as a cottage industry into the institutionalised asset class that exists today. Throughout this period, a number of studies have been conducted on how to evaluate private equity fund managers – the general partners (GPs) – including track record analysis, value creation drivers and deal flow sources. In reality, however, a large number of investors, or limited partners (LPs), are accessing private equity via a service provider, which may be a fund of funds manager, separate account manager or consultant. Evaluating service providers is vastly different from assessing GPs directly. How should their performance be analysed? What is the appropriate benchmark? What are their sources of value add? This chapter attempts to answer those questions.

Performance matters

A number of factors can drive the reported performance of a private equity portfolio. An LP should carefully evaluate the underlying drivers and determine which are spurious and which are likely to persist. As Figure 17.1 shows, private equity has been, on average, the best performing asset class for US state pension plans over the last ten years; it has been a much needed source of alpha for these plans. At the same time, the experience of individual plans has varied widely, as there has been a spread of 780

Figure 17.1: **Tenth to 90th percentile of state fund returns (2003–2012)**
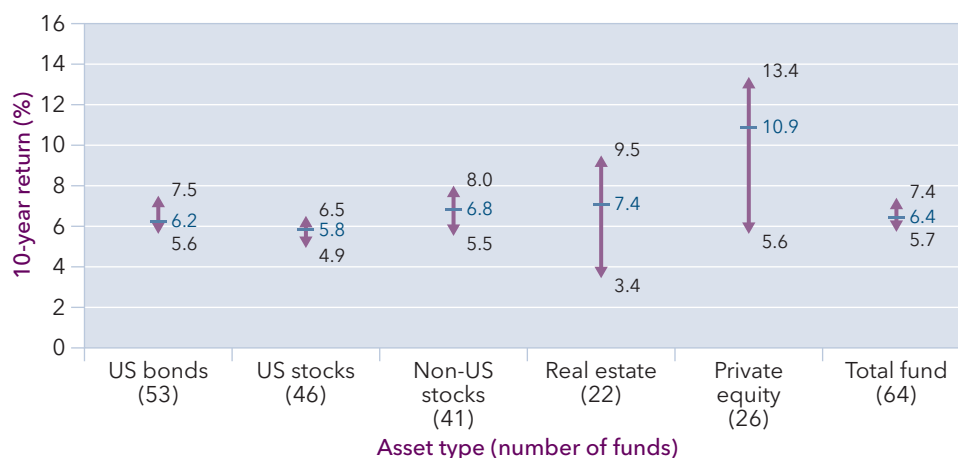
Note: Year ending 30 June.
Source: Cliffwater 2013 Report on State Pension Performance and Trends.

basis points between top and bottom decile performance. Given this wide a range of performance, the service provider's skills in superior investment selection and portfolio construction have a material impact on the plan portfolio's ultimate returns. For example, for a large pension plan with $500 million net asset value in private equity, outperformance of even 100 basis points produces an additional $50 million in value over ten years. That is the kind of impact that matters for the plan. It matters for the beneficiaries.

A head-to-head comparison

In traditional liquid asset classes, the Global Investment Performance Standards (GIPS®) have long served as voluntary, but widely used, performance presentation guidelines for asset managers seeking institutional capital. In 2010, the CFA Institute released the GIPS for private equity. However, private equity GIPS have been slow to catch on, and few managers have undertaken the cumbersome process of adopting them.[1] In the absence of a widely followed standard, private equity service providers, like GPs, will attempt to present their returns in a format that is most favourable to them. The resulting lack of consistency makes performance comparison challenging for an investor.

Consider the following example of two service providers, Redium Capital and Plaudio LP. Both firms have an investment track record spanning more than ten years, and both have generated a since-inception internal rate of return (IRR) of approximately 11 percent (see Table 17.1)[2]. Based on the belief that performance is comparable, the decision to invest may come down to style, reputation or personal preference.

Although performance assessment can be challenging, this process need not begin and end with a single number. It is important to assess what aspects, both within and outside of the service provider's control, have impacted historical performance. Certain aspects, such as consistent selection of outperforming GPs and proactive strategy allocation, are within the service provider's control and indicate skill in investing. Other attributes outside of the service provider's control, such as starting year of the

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Table 17.1: **Comparison of since inception returns between two service providers**

	Redium Capital	Plaudio LP
Year established	1995	2000
Since inception IRR	11.1%	11.9%
Capital committed	$1.6 billion	$2.3 billion

Note: Data as of 30 June 2013.
Source: Hamilton Lane.

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[1] Jacobius, Arleen. Alts managers slow to go with GIPS. *Pensions & Investments*, 1 August 2013.
[2] All data presented in this chapter are current as of 30 June 2013 unless otherwise specified. Vintage years 2011–13 are excluded since they may be largely unfunded and may not yet show meaningful returns.